The ILP Miner

AIS Meeting
S.J. van Zelst
July 17, 2014

Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Presentation goal

Outline

- Preliminaries
  - Prefix closed language
  - Parikh vectors
  - Arc labels
- Language-based theory of regions
  - Place validness
  - Decision variables
  - Definition
- ILP Miner
  - Place expressiveness
  - ILP Formulation
- Challenges
  - Performance
  - Noise
  - Approach
- Questions & Discussion

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Preliminaries

Prefix closed language

- $\mathcal{A} = \{a, b, c, d\}$

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- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$

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Preliminaries

Parikh vectors

- $w = w' \circ l$

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Preliminaries

Parikh vectors

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- $\overrightarrow{w}, \overrightarrow{w}' \in \mathbb{Z}^{\mathcal{A}}$
- $\overrightarrow{w}(a) = 1, \overrightarrow{w}(b) = 2, \overrightarrow{w}(c) = 1$

Preliminaries

Parikh vectors

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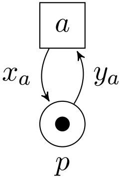
- $\overrightarrow{w}'(a) = 1, \overrightarrow{w}'(b) = 2, \overrightarrow{w}'(c) = 0$

Preliminaries

Arc labels



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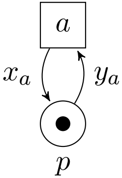
Language-based theory of regions

Place validness



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Language-based theory of regions

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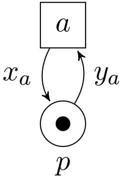

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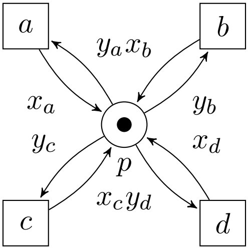

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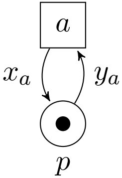

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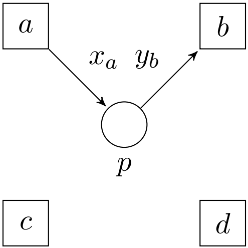

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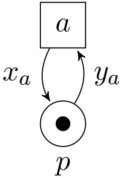

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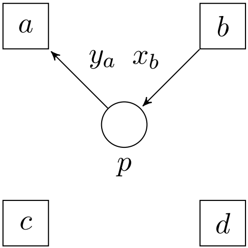
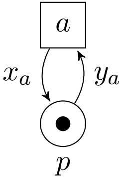


Language-based theory of regions

Decision variables

- $\overrightarrow{x}, \overrightarrow{y} \in \{0,1\}^{\mathcal{A}}$



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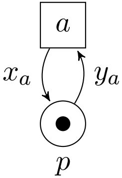
- $\overrightarrow{x}, \overrightarrow{y} \in \{0,1\}^{\mathcal{A}}$
- $\overrightarrow{x}(a) = 1 \implies a \in \bullet p$, $\overrightarrow{x}(a) = 0 \implies a \notin \bullet p$



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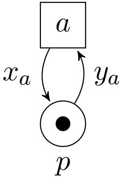

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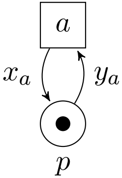

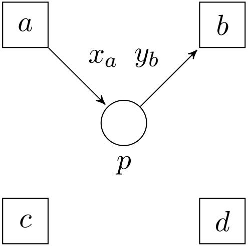

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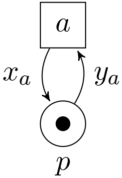

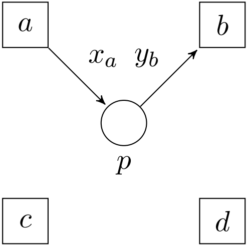

- $\overrightarrow{x}(a) = 1, \overrightarrow{x}(b) = 0, \overrightarrow{x}(c) = 0, \overrightarrow{x}(d) = 0$

Language-based theory of regions

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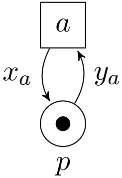

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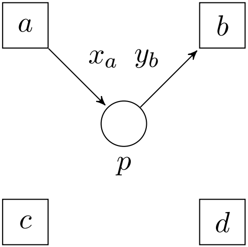

- $\overrightarrow{x}(a) = 1, \overrightarrow{x}(b) = 0, \overrightarrow{x}(c) = 0, \overrightarrow{x}(d) = 0$
- $\overrightarrow{y}(a) = 0, \overrightarrow{y}(b) = 1, \overrightarrow{y}(c) = 0, \overrightarrow{y}(d) = 0$

Language-based theory of regions

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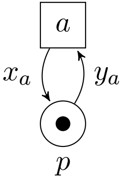

- $\overline{m} \in \{0, 1\}$

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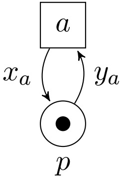

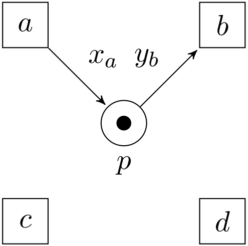

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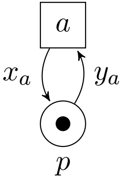

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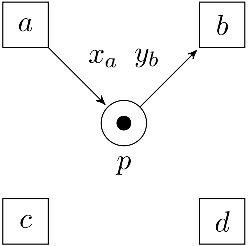
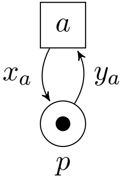


- $\overline{m} = 1$

Language-based theory of regions

Definition

- Triplet assignment $(\overline{m}, \overrightarrow{x}, \overrightarrow{y})$



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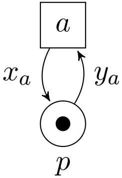

- Triplet assignment $(\overline{m}, \overrightarrow{x}, \overrightarrow{y})$
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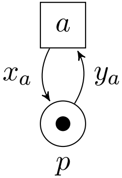

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- $\overline{m} + \sum_{l \in \mathcal{A}} (\overrightarrow{w'}(l) \cdot \overrightarrow{x}(l) - \overrightarrow{w}(l) \cdot \overrightarrow{y}(l)) \geq 0$

Language-based theory of regions

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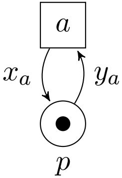

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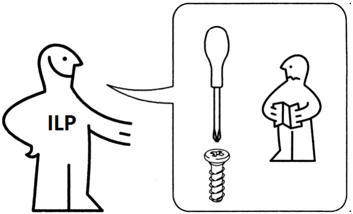

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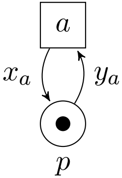

- $\overline{m} + \sum_{l \in \mathcal{A}} (\overrightarrow{w'}(l) \cdot \overrightarrow{x}(l) - \overrightarrow{w}(l) \cdot \overrightarrow{y}(l)) \geq 0$
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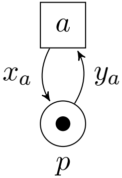

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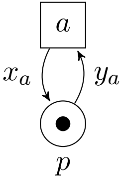
- $w = \epsilon \circ a$
  - $\overline{m} - y_a \geq 0$



Language-based theory of regions

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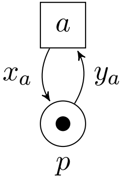
- $w = \langle a \rangle \circ b$
    - $\overline{m} - y_a \geq 0$
    - $\overline{m} - y_a + x_a - y_b \geq 0$



Language-based theory of regions

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- $w = \langle a\rangle \circ c$
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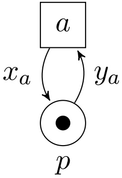

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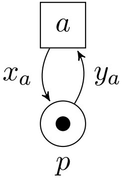

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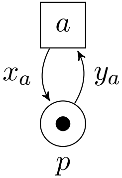

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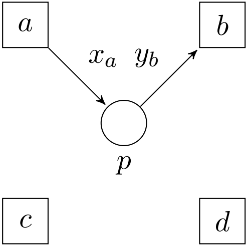

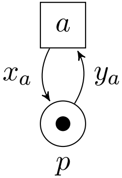

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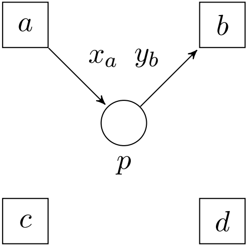

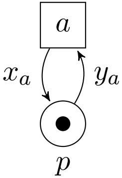

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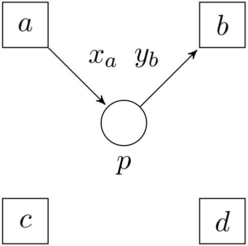

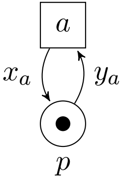

- $0 - 0 \geq 0$ ✓

Language-based theory of regions

Definition

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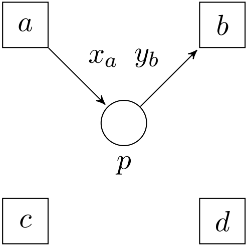

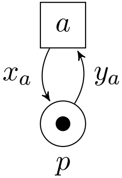

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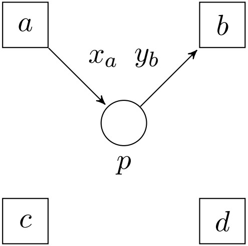

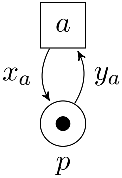

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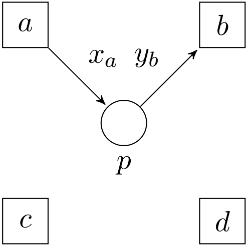

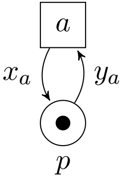

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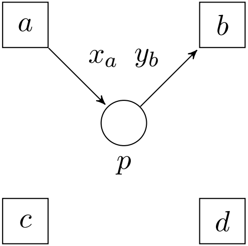

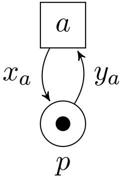

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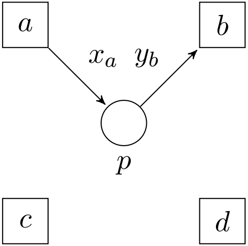

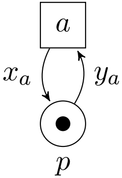

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Language-based theory of regions

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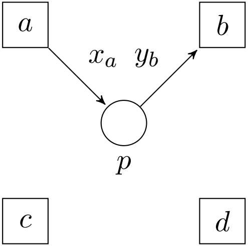

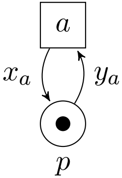

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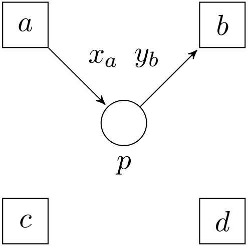

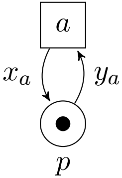

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Language-based theory of regions

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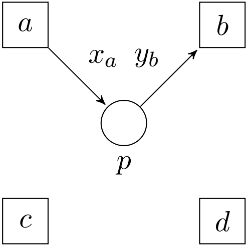

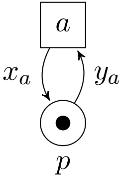

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Language-based theory of regions

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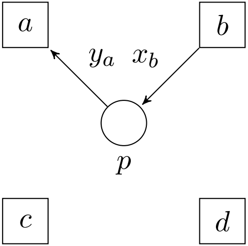

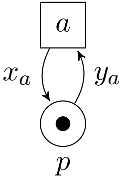

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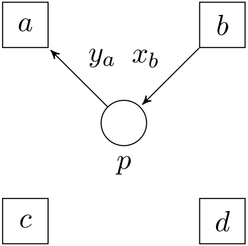

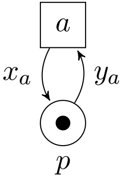

- $\overline{m} - y_a \geq 0$

Language-based theory of regions

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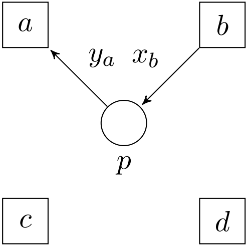

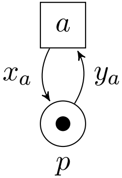

- $0 - 1 \geq 0$ ✗

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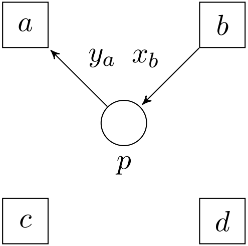

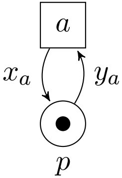

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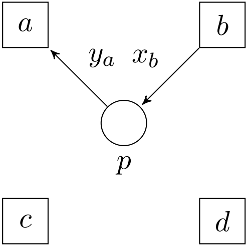

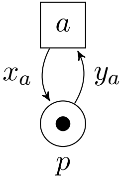

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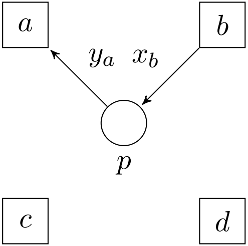

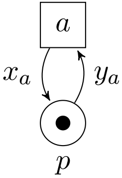

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Language-based theory of regions

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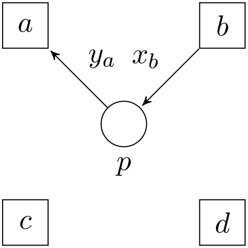

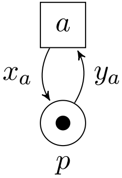

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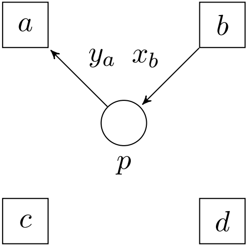

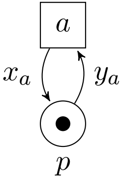

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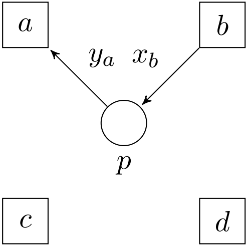

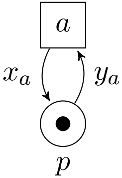

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Language-based theory of regions

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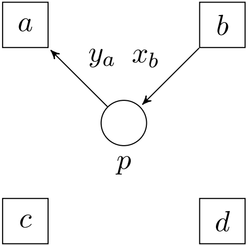

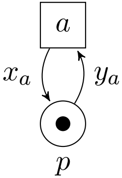

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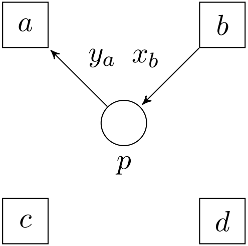

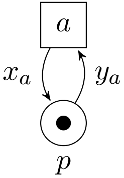

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Language-based theory of regions

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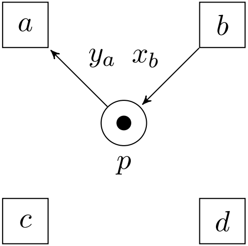

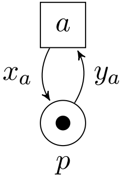

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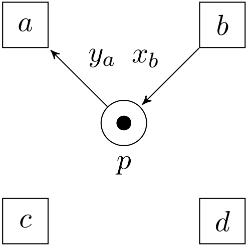

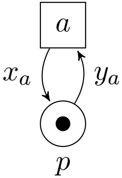

- $\overline{m} - y_a \geq 0$

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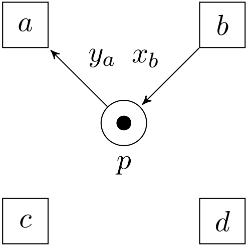

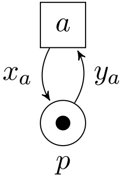

- $1 - 1 \geq 0$ ✓

Language-based theory of regions

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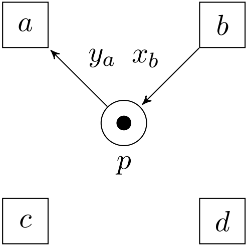

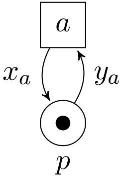

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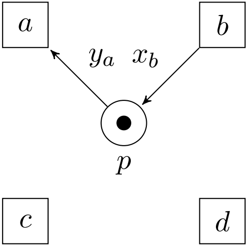

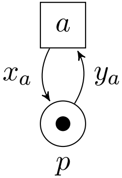

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Language-based theory of regions

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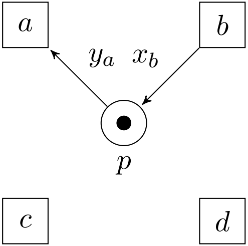

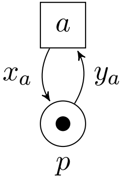

- $1 - 1 \geq 0$ ✓
- $1 - 1 + 0 - 0 \geq 0$ ✓
- $\overline{m} - y_a + x_a - y_c \geq 0$

Language-based theory of regions

Definition

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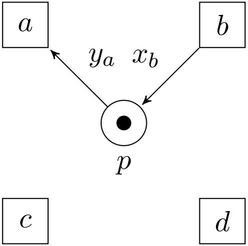

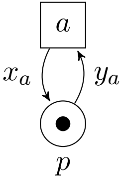

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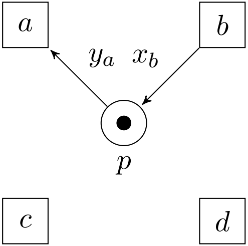

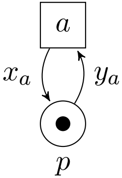

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- $\overline{m} - y_a + x_a - y_b + x_b - y_d \geq 0$

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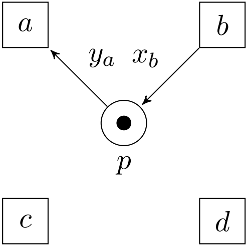

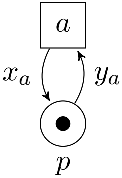

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- $1 - 1 + 0 - 0 \geq 0$ ✓
- $1 - 1 + 0 - 0 + 1 - 0 \geq 0$ ✓

Language-based theory of regions

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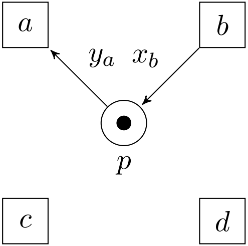

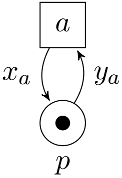

- $1 - 1 \geq 0$ ✓
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- $1 - 1 + 0 - 0 + 1 - 0 \geq 0$ ✓
- $\overline{m} - y_a + x_a - y_c + x_c - y_d \geq 0$

Language-based theory of regions

Definition

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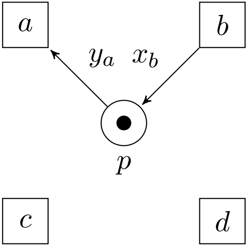
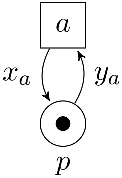


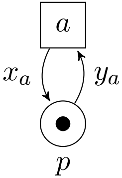


- $1 - 1 \geq 0$ ✓
- $1 - 1 + 0 - 0 \geq 0$ ✓
- $1 - 1 + 0 - 0 \geq 0$ ✓
- $1 - 1 + 0 - 0 + 1 - 0 \geq 0$ ✓
- $1 - 1 + 0 - 0 + 0 - 0 \geq 0$ ✓

Outline

ILP Miner

Place expressivenes

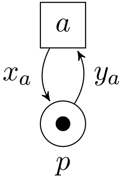


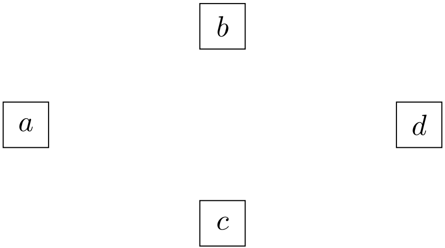
- $\mathcal{L}_r = \{\langle a, b, d \rangle, \langle a, c, d \rangle\}$

ILP Miner

Place expressivenes

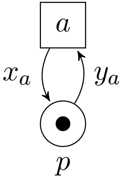
- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$



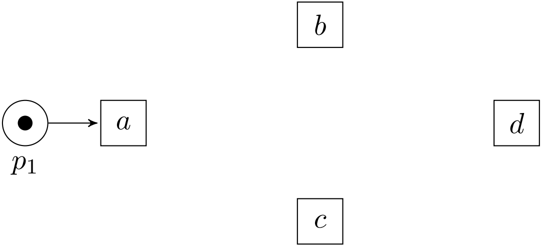


ILP Miner

Place expressivenes



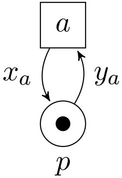
- $\mathcal{L}_r = \{\langle a, b, d \rangle, \langle a, c, d \rangle\}$



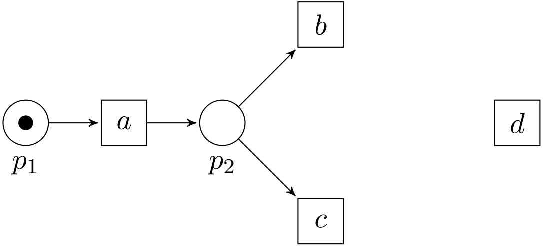
- $\overline{m} = 1, y_a = 1$ ✓

ILP Miner

Place expressivenes

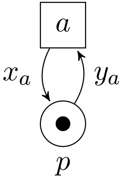


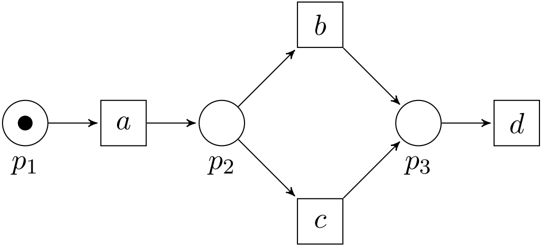
- $\mathcal{L}_r = \{\langle a, b, d \rangle, \langle a, c, d \rangle\}$



- $\overline{m} = 0, x_a = 1, y_b = 1, y_c = 1$ ✓

- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$



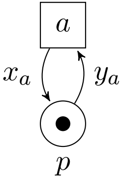


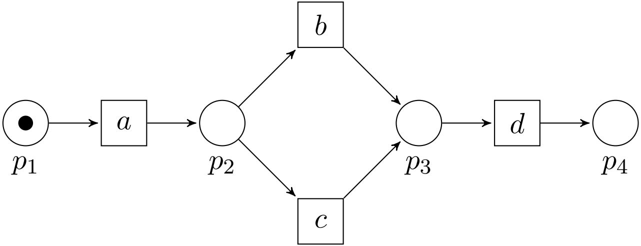
- $\overline{m} = 0, x_b = 1, x_c = 1, y_d = 1$ ✓

ILP Miner

Place expressivenes

- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$



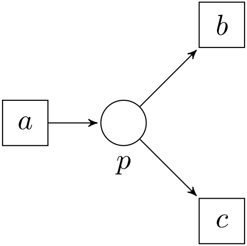


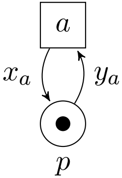
- $\overline{m} = 0, x_d = 1$ ✓

ILP Miner

Place expressivenes

- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$

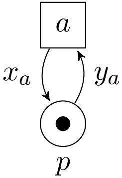




- $\overline{m} = 0, x_a = 1, y_b = 1, y_c = 1$ ✓

ILP Miner

Place expressivenes



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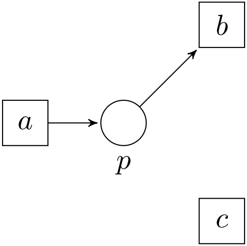
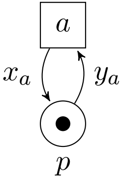


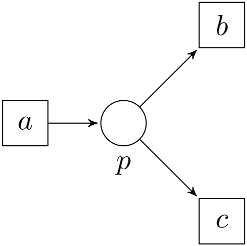
- $\overline{m} = 0, x_a = 1, y_b = 1$ ✓

ILP Miner

Place expressivenes

- $\mathcal{L}_r = \{\langle a, b, c, d\rangle, \langle a, c, b, d\rangle\}$



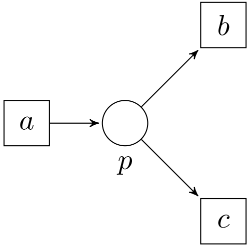


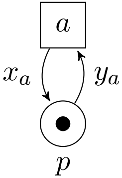
- $\overline{m} = 0, x_a = 1, y_b = 1, y_c = 1$?

ILP Miner

Place expressivenes

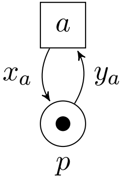
- $\mathcal{L}_r = \{\langle a, b, c, d\rangle, \langle a, c, b, d\rangle\}$

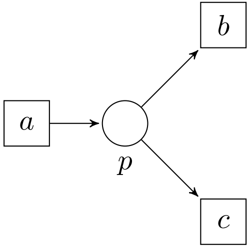




- $\overline{m} = 0, x_a = 1, y_b = 1, y_c = 1$?
- $\langle a, b, c\rangle \rightarrow \overline{m} - y_a + x_a - y_b + x_b - y_c \geq 0$

- $\mathcal{L}_r = \{\langle a, b, c, d\rangle, \langle a, c, b, d\rangle\}$





- $\overline{m} = 0, x_a = 1, y_b = 1, y_c = 1$ **?**
- $\langle a, b, c\rangle \rightarrow \overline{m} - y_a + x_a - y_b + x_b - y_c \geq 0$
- $\langle a, b, c\rangle \rightarrow 0 - 0 + 1 - 1 + 0 - 1 \geq 0$

ILP Miner

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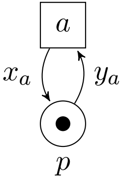

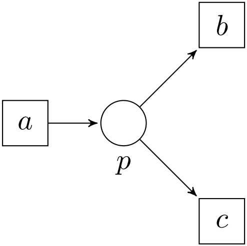

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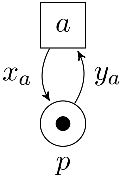
- $\langle a, b, c\rangle \rightarrow \overline{m} - y_a + x_a - y_b + x_b - y_c \geq 0$
- $\langle a, b, c\rangle \rightarrow 0 - 0 + 1 - 1 + 0 - 1 \geq 0$ ✗

ILP Miner

Place expressivenes



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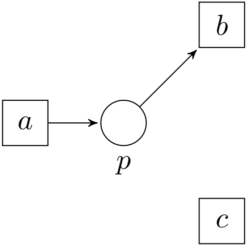
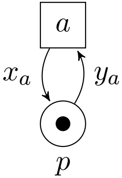
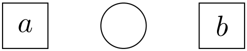


- $\overline{m} = 0, x_a = 1, y_b = 1$ ✓

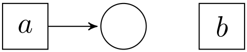
ILP Miner

Place expressivenes



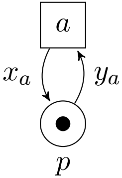


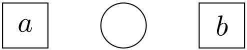
- vs.



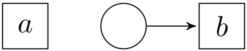
ILP Miner

Place expressiveness



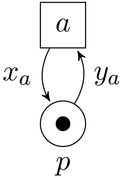


- vs.

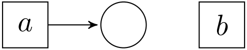


ILP Miner

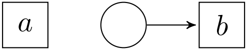
Place expressiveness



- Minimize:



- Maximize:



ILP Miner

ILP Formulation

- M, M' are matrices of size $|\mathcal{L}_p \setminus \{\epsilon\}| \times |\mathcal{A}|$

ILP Miner

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- $M(w, l) = \overrightarrow{w}(l)$, $M'(w, l) = \overrightarrow{w}'(l)$

ILP Miner

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ILP Miner

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- M, M' are matrices of size $|\mathcal{L}_p \setminus \{\epsilon\}| \times |\mathcal{A}|$
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- $\mathcal{L}_p = \{\epsilon, \langle a \rangle, \langle a,b \rangle, \langle a,c \rangle, \langle a,b,d \rangle, \langle a,c,d \rangle\}$

$$M = \begin{array}{c} \\ \langle a \rangle \\ \langle a,b \rangle \\ \langle a,c \rangle \\ \langle a,b,d \rangle \\ \langle a,c,d \rangle \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

ILP Miner

ILP Formulation

- M, M' are matrices of size $|\mathcal{L}_p \setminus \{\epsilon\}| \times |\mathcal{A}|$
- $M(w,l) = \overrightarrow{w}(l)$, $M'(w,l) = \overrightarrow{w}'(l)$
- $\mathcal{L}_p = \{\epsilon, \langle a \rangle, \langle a,b \rangle, \langle a,c \rangle, \langle a,b,d \rangle, \langle a,c,d \rangle\}$

$$M' = \begin{array}{c} \\ \langle a \rangle \\ \langle a,b \rangle \\ \langle a,c \rangle \\ \langle a,b,d \rangle \\ \langle a,c,d \rangle \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

ILP Miner

ILP Formulation

- $\overline{m} + \sum_{l \in \mathcal{A}} (\overrightarrow{w'}(l) \cdot \overrightarrow{x}(l) - \overrightarrow{w}(l) \cdot \overrightarrow{y}(l)) \geq 0$

ILP Miner

ILP Formulation

- $\overline{m} + \sum_{l \in \mathcal{A}} (\overrightarrow{w'}(l) \cdot \overrightarrow{x}(l) - \overrightarrow{w}(l) \cdot \overrightarrow{y}(l)) \geq 0$

$$=$$

ILP Miner

ILP Formulation

- $\overline{m} + \sum_{l \in \mathcal{A}} (\overrightarrow{w'}(l) \cdot \overrightarrow{x}(l) - \overrightarrow{w}(l) \cdot \overrightarrow{y}(l)) \geq 0$

$$=$$

- $\overline{m} \cdot \overrightarrow{1} + M' \cdot \overrightarrow{x} - M \cdot \overrightarrow{y} \geq \overrightarrow{0}$

ILP Miner

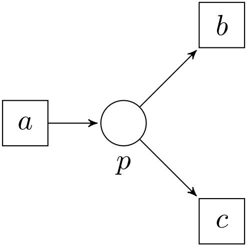
ILP Formulation

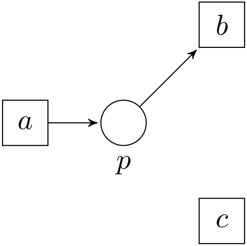
- $\mathcal{L}_p = \{\epsilon, \langle a \rangle, \langle a, b \rangle, \langle a, c \rangle, \langle a, b, d \rangle, \langle a, c, d \rangle\}$

$$\begin{array}{l} \\ \langle a \rangle \\ \langle a, b \rangle \\ \langle a, c \rangle \\ \langle a, b, d \rangle \\ \langle a, c, d \rangle \end{array} \begin{array}{c} \begin{array}{ccccccccccc} \overline{m} & x_a & x_b & x_c & x_d & y_a & y_b & y_c & y_d & & \end{array} \\ \left(\begin{array}{ccccccccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \geq & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & \geq & 0 \\ 1 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & \geq & 0 \\ 1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & \geq & 0 \\ 1 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & -1 & \geq & 0 \end{array} \right) \end{array}$$

ILP Miner

ILP Formulation





ILP Miner

ILP Formulation

- $\mathcal{L}_r = \{\langle a, b, d\rangle, \langle a, c, d\rangle\}$
- $\mathcal{L}_p = \{\epsilon, \langle a\rangle, \langle a, b\rangle, \langle a, c\rangle, \langle a, b, d\rangle, \langle a, c, d\rangle\}$

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- $\mathcal{L}_p = \{\epsilon, \langle a\rangle, \langle a, b\rangle, \langle a, c\rangle, \langle a, b, d\rangle, \langle a, c, d\rangle\}$
- $\texttt{count}(a \in \mathcal{L}_p) = \texttt{count}(b \in \mathcal{L}_p) + \texttt{count}(c \in \mathcal{L}_p) + 1$

ILP Miner

ILP formulation

Minimize	$\overline{m} + \overrightarrow{1}^T \cdot (\overline{m} \cdot \overrightarrow{1} + M \cdot (\overrightarrow{x} - \overrightarrow{y}))$	Minimal and most expressive regions
such that	$\overline{m} \cdot \overrightarrow{1} + M' \cdot \overrightarrow{x} - M \cdot \overrightarrow{y} \geq \overrightarrow{0}$	Theory of regions
	$\overrightarrow{1}^T \cdot \overrightarrow{x} + \overrightarrow{1}^T \cdot \overrightarrow{y} \geq 1$	Avoid trivial minimal region $(0, \overrightarrow{0}, \overrightarrow{0})$
	$\overrightarrow{0} \leq \overrightarrow{x} \leq \overrightarrow{1}$	$\overrightarrow{x} \in \{0, 1\}^{\mathcal{A}}$
	$\overrightarrow{0} \leq \overrightarrow{y} \leq \overrightarrow{1}$	$\overrightarrow{y} \in \{0, 1\}^{\mathcal{A}}$
	$0 \leq \overline{m} \leq 1$	$\overline{m} \in \{0, 1\}$

ILP Miner

ILP formulation

- $\mathcal{L}_p = \{\epsilon, \langle a \rangle, \langle a, b \rangle, \langle a, c \rangle, \langle a, b, d \rangle, \langle a, c, d \rangle\}$

$$
\begin{array}{l}
 \\
min : \\
\langle a \rangle \\
\langle a, b \rangle \\
\langle a, c \rangle \\
\langle a, b, d \rangle \\
\langle a, c, d \rangle \\
\neg(0, \overrightarrow{0}, \overrightarrow{0})
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc}
\overline{m} & x_a & x_b & x_c & x_d & y_a & y_b & y_c & y_d & & \end{array} \\
\left(\begin{array}{ccccccccccc}
6 & 5 & 2 & 2 & 2 & -5 & -2 & -2 & -2 & & \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \geq & 0 \\
1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & \geq & 0 \\
1 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & \geq & 0 \\
1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & \geq & 0 \\
1 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & -1 & \geq & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \geq & 1
\end{array}\right)
\end{array}
$$

- $\mathcal{L}_p = \{\epsilon, \langle a \rangle, \langle a,b \rangle, \langle a,c \rangle, \langle a,b,d \rangle, \langle a,c,d \rangle\}$
- $a \rightarrow b$

$$
\begin{array}{l}
\\
min: \\
\langle a \rangle \\
\langle a,b \rangle \\
\langle a,c \rangle \\
\langle a,b,d \rangle \\
\langle a,c,d \rangle \\
\neg(0,\vec{0},\vec{0}) \\
a \rightarrow b
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccc}
\overline{m} & x_a & x_b & x_c & x_d & y_a & y_b & y_c & y_d & & \\
\end{array} \\
\left(\begin{array}{ccccccccccc}
6 & 5 & 2 & 2 & 2 & -5 & -2 & -2 & -2 & & \\
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \geq & 0 \\
1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & \geq & 0 \\
1 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & \geq & 0 \\
1 & 1 & 1 & 0 & 0 & -1 & -1 & 0 & -1 & \geq & 0 \\
1 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & -1 & \geq & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \geq & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & = & 2
\end{array}\right)
\end{array}
$$

Outline

Preliminaries
- Prefix closed language
- Parikh vectors
- Arc labels

Language-based theory of regions
- Place validness
- Decision variables
- Definition

ILP Miner
- Place expressiveness
- ILP Formulation

Challenges
- Performance
- Noise
- Approach

Questions & Discussion

Challenges

Performance

Challenges

Performance

- $2 \cdot |\mathcal{A}| + 1$ variables

Challenges

Performance

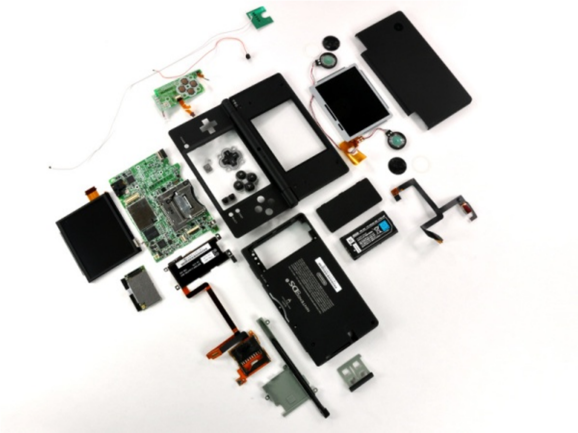
- $2 \cdot |\mathcal{A}| + 1$ variables
- $\pm|\mathcal{L}_p|$ constraints

Challenges

Noise

Challenges

Approach



Challenges

Approach

Challenges

Approach

Outline

Questions & Discussions